INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, Second Semester, 2020-21
Statistics - II, Semesteral Examination, May 18, 2021

Answer all questions **Maximum Marks: 50**

Time: $2\frac{1}{2}$ hours; submission must be complete by 1 pm

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You may freely consult the lecture notes, but no other books or resources may be consulted. You may use any of the results stated and discussed in the lecture notes, by stating them explicitly. Results from the assignments may not be used without establishing them.

1. Consider a random sample $X_1, X_2, \ldots, X_n$ from a population with density $f(x|\theta) = \exp(-(x-\theta))$, for $x > \theta$ where $\theta > 0$ is an unknown parameter.

(a) Find the minimal sufficient statistics T for θ.

(b) Assuming that T is complete, find the UMVUE of θ.

(c) Let $S = \sum_{i=1}^{n}(X_i - \bar{X})^2$. Show that S and T are independent random variables. [4+4+2]

2. Let $X \sim \text{Poisson}(\lambda)$, $\lambda > 0$. Define $Y = 1$ when $X > 0$, and 0 otherwise.

(a) Find the Fisher information on λ (say, $I^{(X)}(\lambda)$ and $I^{(Y)}(\lambda)$, respectively) contained in X and Y individually.

Consider a random sample $X_1, X_2, \ldots, X_n$ from the X population and the corresponding $Y_1, Y_2, \ldots, Y_n$.
(b) Construct consistent estimators $T_1(X_1, X_2, \ldots, X_n)$ and $T_2(Y_1, Y_2, \ldots, Y_n)$ for λ based on the MLE of λ from the two data sets.

(c) Find the asymptotic distributions of T_1 and T_2.

(d) Find the asymptotic relative efficiency of of T_1 w.r.t. T_2. [5+5+6+2]

3. Suppose $X \sim P_\theta$ with probability mass function p_θ. It is of interest to test $H_0 : \theta = \theta_0$ versus $H_1 : \theta = \theta_1$ at the significance level $\alpha = 0.05$. For $i = 1, 2$, p_{θ_i} is as follows:

x	1	2	3	4	5
$p_{\theta_0}(x)$	.015	.080	.800	.090	.015
$p_{\theta_1}(x)$	.100	.100	.600	.100	.100

(a) Find the most powerful test for testing H_0 versus H_1 at the significance level of $\alpha = 0.05$.

(b) Find the probabilities of Type I and Type II errors for this test. [7+3]

4. In an ecological study 5 independent attempts were made to photographically capture (or to camera trap) a particular tiger. The fourth attempt provided the only success. The success probability, θ, is known as the detection probability. Assume that the prior distribution on θ is Beta(0.3, 1.2).

(a) Derive the posterior distribution of θ given the data.

(b) Explain how the 95% HPD credible interval for θ can be constructed.

(c) Consider testing $H_0 : \theta \leq 0.25$ versus $H_1 : \theta > 0.25$. Explain the Bayesian approach for this. [4+4+4]